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A study for evaluation method of viscoelastic materials under dynamic loading

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Abstract

Indirect impact method using fixed end reflection was newly proposed as the technique for evaluating the dynamic characteristic of viscoelastic materials. A cylindrical specimen is sandwiched between a viscoelastic input bar and a rigid wall, and is tested impacting the input bar. The reflection and the transmission at interfaces as well as the longitudinal wave propagation in viscoelastic bars were analyzed based on the elementary theory in the frequency domain. Some kinds of viscoelastic materials were tested using the proposed method, and the dynamic properties of the materials were evaluated within the frequency of about 15kHz. Moreover, the validity of the proposed method was examined by comparing with the conventional methods. Consequently, it is shown that the proposed method could easily decide the characteristic values of various materials in the elastic region with high accuracy.

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Keywords: Viscoelastic properties; Stress wave; Impact; Damping; Attenuation; Dispersion;

1. Introduction

Polymer materials have been widely used in applications requiring lightness and high damping capacity. Such materials show the viscoelastic behaviour at high strain rates. It is, therefore, important to grasp the characteristics of materials under dynamic loading. The dynamic properties of viscoelastic materials have been evaluated by some techniques such as the wave propagation method [1-3] and the viscoelastic SHB method [4-6]. The wave propagation method is used a long specimen to check directly the waveform

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Nomenclature

A	cross-sectional area
B	terms of waveform change
c	propagation velocity
D	diameter
E	Young's modulus
i	imaginary unit
J	complex compliance
k	wave number
l	length
t	time
x	coordinate along rod axis

Greek letters

α	attenuation coefficient
ε	strain
η	viscosity of dashpot in 3-element solid model
ρ	density
σ	stress
ω	angular frequency

Subscripts

A	interface A
B	interface B
i	incident
r	reflected
s	specimen
t	transmitted
1	real part of complex compliance
2	imaginary part of complex compliance
①	first position of strain gage
②	second position of strain gage
I	first bar
II	second bar

change of stress waves propagating in the specimen. The accuracy of the method is comparatively high, but the specimens could not be prepared easily according to kinds of material. On the other hand, the viscoelastic SHB method consists of holding a small specimen between the input and the output bars made of viscoelastic materials. It determines the relation of the stress and the strain of the specimen indirectly from the measured waves on the input/output bars. However errors are generally supposed to be larger due to aspect ratio of the specimens or impedance deference between specimen and input/output bars. The method for evaluating the dynamic properties of viscoelstic materials has not been established. In this work, a newly indirect method using fixed end reflection is proposed. A cylindrical specimen is sandwiched between a viscoelastic input bar made of polymethyl methacrylate (PMMA) and a metallic wall. and is tested indirectly impacting the input bar. Some kinds of viscoelastic materials are tested, and are determined the viscoelastic characteristics by identifying as a 3-element solid model.

2. One-dimensional wave propagation in viscoelastic bar

2.1. Strain wave in half infinite bar

Consider a thin and uniform viscoelastic bar, let $\bar{\varepsilon}(x,\omega)$ be the Fourier transform of a strain-time relation $\varepsilon(x,t)$. When the material is linear viscoelastic, the following equation can be found [2]:

$$\bar{\varepsilon}(x,\omega) = \bar{\varepsilon}(0,\omega)\exp\{(-\alpha + ik)x\}. \tag{1}$$

where ω and i are angular frequency and imaginary unit, respectively. The attenuation coefficient α and the wave number k are related to the complex compliance $J(\omega)$ as

$$\left.\begin{aligned} k^2 - \alpha^2 &= \rho\omega^2 J_1(\omega) \\ 2\alpha k &= \rho\omega^2 J_2(\omega) \end{aligned}\right\}, \tag{2}$$

where ρ is the mass density. The complex compliance, which shows material properties of a viscoelastic medium, is expressed by the ratio of strain $\bar{\varepsilon}(\omega)$ to stress $\bar{\sigma}(\omega)$ in the frequency domain as follows:

$$J(\omega) = J_1(\omega) - iJ_2(\omega) = \frac{\bar{\varepsilon}(\omega)}{\bar{\sigma}(\omega)}, \tag{3}$$

where $J_1(\omega)$ and $J_2(\omega)$ are the real and the imaginary part of the complex compliance.

2.2. Strain wave at interface

Consider uniform elastic bars I and II which contact at the interface $x = 0$ as shown in Fig.1. When the incident stress wave $\sigma_i(t)$ reaches the interface $x = 0$, some part of wave is reflected to the medium I as reflected wave $\sigma_r(t)$ and the rest is transmitted to the medium II as transmitted wave $\sigma_t(t)$. Assuming the force equilibrium and continuity of particle velocity at the interface, $\sigma_r(t)$ and $\sigma_t(t)$ are obtained in terms of $\sigma_i(t)$ as the following forms:

$$\sigma_r(t) = \frac{A_{\mathrm{II}}E_{\mathrm{II}}c_{\mathrm{I}} - A_{\mathrm{I}}E_{\mathrm{I}}c_{\mathrm{II}}}{A_{\mathrm{II}}E_{\mathrm{II}}c_{\mathrm{I}} + A_{\mathrm{I}}E_{\mathrm{I}}c_{\mathrm{II}}}\sigma_i(t), \tag{4}$$

$$\sigma_t(t) = \frac{2A_{\mathrm{I}}E_{\mathrm{I}}c_{\mathrm{I}}}{A_{\mathrm{II}}E_{\mathrm{II}}c_{\mathrm{I}} + A_{\mathrm{I}}E_{\mathrm{I}}c_{\mathrm{II}}}\sigma_i(t), \tag{5}$$

where A_{I}, E_{I}, ρ_{I} and c_{I} represent the cross section, Young's modulus, the material density and the

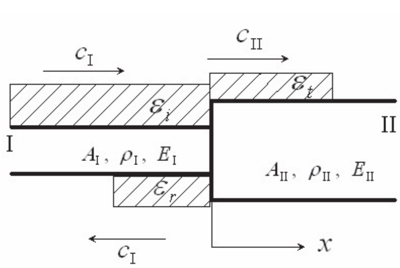


Fig.1 Reflection and transmission at interface

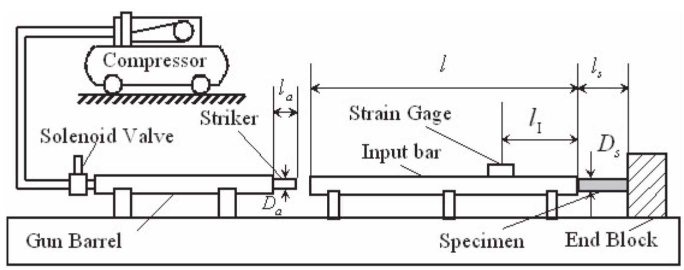


Fig.2 Experimental apparatus of indirect method using fixed end reflection

wave velocity correspondent to the medium I. Similarly, A_{II}, E_{II}, ρ_{II}, c_{II} correspond to the medium II. The Fourier transformation is applied to Eqs.(4) and (5). Then, replacing E by $1/J(\omega)$ according to the correspondence principle and using Eq.(3), the incident and reflected strain waves for a viscoelastic medium in the frequency domain can be obtained as the following equations:

$$\bar{\varepsilon}_r(\omega) = \frac{A_{\mathrm{II}}\sqrt{\rho_{\mathrm{II}}J_{\mathrm{I}}(\omega)} - A_{\mathrm{I}}\sqrt{\rho_{\mathrm{I}}J_{\mathrm{II}}(\omega)}}{A_{\mathrm{II}}\sqrt{\rho_{\mathrm{II}}J_{\mathrm{I}}(\omega)} + A_{\mathrm{I}}\sqrt{\rho_{\mathrm{I}}J_{\mathrm{II}}(\omega)}}\bar{\varepsilon}_i(\omega)\ . \tag{6}$$

$$\bar{\varepsilon}_t(\omega) = \frac{2A_{\mathrm{I}}\sqrt{\rho_{\mathrm{II}}J_{\mathrm{I}}(\omega)}}{A_{\mathrm{II}}\sqrt{\rho_{\mathrm{II}}J_{\mathrm{I}}(\omega)} + A_{\mathrm{I}}\sqrt{\rho_{\mathrm{I}}J_{\mathrm{II}}(\omega)}}\frac{J_{\mathrm{II}}(\omega)}{J_{\mathrm{I}}(\omega)}\bar{\varepsilon}_i(\omega)\ . \tag{7}$$

Taking $A_{\mathrm{II}} \to \infty$ makes $\bar{\varepsilon}_r(\omega) = \bar{\varepsilon}_i(\omega)$ for the fixed end. Fractions in right sides of Eqs.(6) and (7) show the coefficients of reflection $R = \bar{\varepsilon}_r(\omega)/\bar{\varepsilon}_i(\omega)$ and the coefficients of transmission $T = \bar{\varepsilon}_t(\omega)/\bar{\varepsilon}_i(\omega)$.

3. Indirect impact method using fixed end reflection

3.1. Measurement Principle

The experimental apparatus of indirect impact method using fixed end reflection is shown schematically in Fig.2. A specimen is held between the input bar and the rigid wall. A compressive wave generated by the impact of a striker bar propagates along the input bar, and is measured by the strain gage situated on the input bar. The compressive wave is reflected at two interfaces, (first; between the input bar and the specimen, second; between the specimen and the rigid wall), and is measured reflected waves.

Figure 3 shows the procedure for predicting waveform at the gage position in frequency domain. Using Eq.(1), the terms of waveform change after propagating in the distance l_{I} and $2l_s$ are given by

$$\left.\begin{aligned} B_{\mathrm{I}} &= \exp\{-(\alpha_{\mathrm{I}} + ik_{\mathrm{I}})l_{\mathrm{I}}\} \\ B_s &= \exp\{-(\alpha_s + ik_s)2l_s\} \end{aligned}\right\} \tag{8}$$

Let the reflectivity and transmittance when a strain wave propagates from the input bar to the specimen be R and T, and let them when a strain wave propagates from the specimen to the input bar be R' and T', respectively. A compressive wave at the gage position in the frequency region assumed to be $\bar{\varepsilon}(\omega)$ as shown in Fig.3. The wave reaches the right end of the input bar as $B_{\mathrm{I}}\bar{\varepsilon}$, and part of it is reflected as $-RB_{\mathrm{I}}\bar{\varepsilon}$ and the rest is transmitted as $TB_{\mathrm{I}}\bar{\varepsilon}$. The reflected wave $-RB_{\mathrm{I}}\bar{\varepsilon}$ will propagate in the distance l_{I}, and become $-RB_{\mathrm{I}}{}^2\bar{\varepsilon}$ at the gage position. If a strain wave reflects totally at the fixed end, the transmitted wave $TB_{\mathrm{I}}\bar{\varepsilon}$ propagates and returns to the same position as $TB_{\mathrm{I}}B_s\bar{\varepsilon}$. Some part of it is reflected and the rest is transmitted. Thus, the wave propagates repeating the reflection and transmission. Consequently, after propagating enough, the superposed wave signal $\bar{\varepsilon}_{gage2}(\omega)$ measured by the strain gage can be calculated as following form.

$$\bar{\varepsilon}_{gage}(\omega) = \bar{\varepsilon}(\omega) - RB_{\mathrm{I}}{}^2\bar{\varepsilon}(\omega) + \sum_{m=1}^{\infty}\left\{B_{\mathrm{I}}{}^2B_sT'TR^{m-1}\right\}\bar{\varepsilon}(\omega) \tag{9}$$

Applying the Fourier inverse transformation to Eq.(9), the strain waveform in the time domain $\bar{\varepsilon}_{gage2}(t)$ can be obtained. It is possible for this method to experiment at higher strain rate than the devised direct method. Moreover, the superposed wave at the center of the specimen $\bar{\varepsilon}_{spe}(\omega)$ is written by

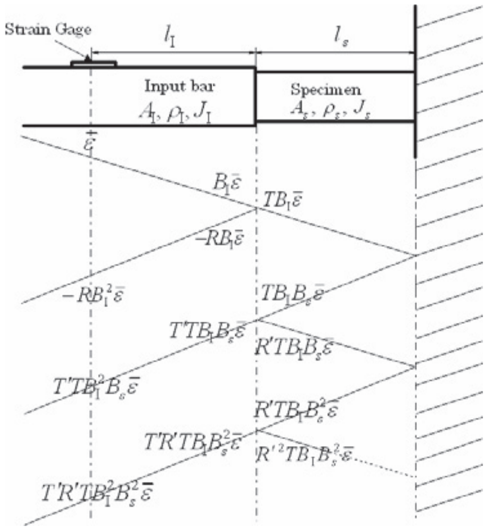


Fig.3 Procedure for strain wave propagating in input bar and specimen in frequency domain

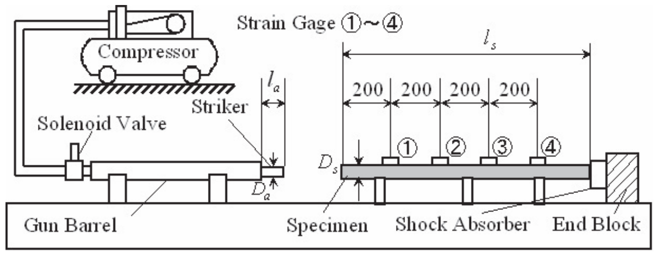


Fig.4 Experimental apparatus of wave propagation method

Table 1 Dimensions of specimen and the striker bar

Material	Specimen		Striker bar	
	Diameter	Length	Diameter	Length
Polyethylene	15	1000	15	50

$$\bar{\varepsilon}_{spe}(\omega) = TB_I \sum_{m=1}^{\infty} \left\{ R'^{m-1} \left(B_s^{\ m-1} + B_s^{\ m} \right) \right\} \bar{\varepsilon}(\omega) \tag{10}$$

3.2. Examination of accuracy

The accuracy of determined viscoelastic properties is examined by comparing the newly devised method with the wave propagation method [2] using the same materials. Polyethylene (PE), which is produced by Koei Industrial Co. ltd, is used as a typical example. The material density is 9.10×10^2 kg/m^3. The schematic drawing of wave propagation method is given in Fig.4. Four strain gages are situated at positions separated by equal intervals (200mm) at a distance from the impact end. The striker bar is launched by the air compressor, and impacts the front end of the specimen bar. The dimensions of the specimen and the striker bar are shown in Table 1. The specimen is identified as a 3-element solid model. The real part and imaginary part of the complex compliance are expressed as follows.

$$J_1^{\ *}(\omega) = \frac{1}{E_1} + \frac{E_2}{E_2^{\ 2} + (\omega\eta_2)^2} \ , \ J_2^{\ *}(\omega) = \frac{\omega\eta_2}{E_2^{\ 2} + (\omega\eta_2)^2} \ . \tag{11}$$

Figure 5 indicates the experimental and the predicted strain histories in the wave propagation method. The predicted waveforms were obtained from the first experimental waveform using Eq. (1) and are shown as the solid line. The maximum strain rate, which is calculated by applying numerical differentiation of the experimental waves, is also written in the figure. It is found that the peak value decreases while the duration time increases as the waves propagate. The experimental and the predicted waveforms coincide with each other. It is verified that the specimen can be accurately determined the viscoelastic properties. The experimental apparatus of new devised method is already shown in Fig. 1. The dimensions of the specimen, the input and the striker bars are denoted in Table 2. The aspect ratio of the specimen is 0.5. The input bar is made of PMMA. Preliminary wave propagation method was conducted to identify a mechanical model for the PMMA bar using a 3-element solid model, $E_1 = 5.44$ GPa , $E_2 = 3.59 \times 10^1$ GPa and $\eta_2 = 2.49$ MPa・s. The experimental and the predicted strain histories including the maximum strain rate in the new devised method are drawn in Fig.6-(a). The

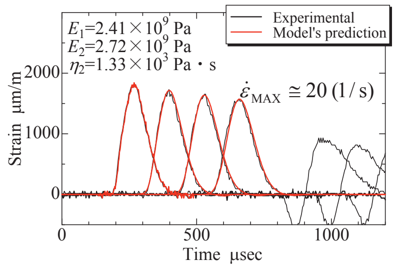
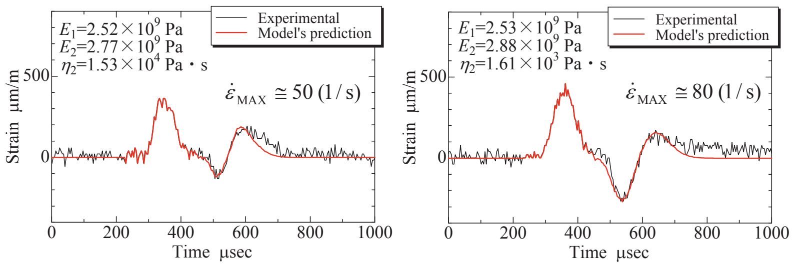


Fig.5 Experimental and predicted waves in wave propagation method (PE)



(a) Length of specimen : 7.5mm (b) Length of specimen : 30mm

Fig.6 Experimental and predicted waves in indirect method using fixed end reflection (PE)

Table 2 Dimensions of specimen, input and striker bars

Material	Specimen		Material	Input bar			Striker bar	
	Diameter	Length		Diameter	Length	l_1	Diameter	Length
Polyethylene	15	7.5	Polymethyl methacrylate	20	1000	200	15	50

experimental waveforms are consistent with the predicted waves obtained by Eq. (9). The parameters in Fig.5 and Fig.6-(a) are almost of the same values at the strain rate of about 10^1 s^{-1}. Consequently, the new devised method can be evaluated the dynamic properties of viscoelastic materials at high accuracy.

3.3. Influence of aspect ratio

The same material PE with a length of 30mm and a diameter of 15mm is tested in the new devised method. The aspect ratio is 2. The experimental and the predicted waveforms in the new devised method are indicated in Fig.6-(b). It can be seen the correspondence between these waves well. Comparing Fig. X with Fig. X (aspect ratio : 0.5), the both characteristic values are also almost identical. Therefore, the new devised method can evaluate the material properties in various aspect ratios of specimens.

4. Conclusions

・The accuracy of the indirect method using fixed end reflection and the conventional method (wave propagation method) were compared by testing the same materials. As a result, it was clarified to obtain the equal accuracy in both methods at the strain rate of about 10^1 s^{-1}.
・It was shown that the aspect ratio of the specimen (from 0.5 to 2) did not influence the accuracy in the indirect method using fixed end reflection.

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